

## Singapore Math

**First**, you need to know that Singapore Math takes a slightly different mathematic approach than what you may be used to. It revolves around several key number-sense strategies:

- (1) building number sense through part-whole thinking,
- (2) understanding place value, and
- (3) breaking numbers into decomposed parts or friendlier numbers, ones that are easier to work with in the four operations (addition, subtraction, multiplication and division).

**Second**, Singapore Math does something dramatically different when it comes to word problems. It relies on model drawing, which uses units to visually represent a word problem. Students learn to visualize what a word problem is saying so they can understand the meaning and thus how to solve the problem.

**Third**, we have mental math, which teaches students to calculate in their heads without using paper and pencil. Sure, your students will still need to commit some facts to memory, but mental math will teach him or her to do calculations using proven strategies that don't require pencil and paper.

**Fourth**, the strategies taught in Singapore are layered upon one another. One strategy is the foundation for another one. For example, students need prior knowledge of bonding in order to be successful at strategies they will learn later on (like vertical addition).

**Last**, Singapore Math teaches students to understand math in stages, beginning With **Concrete** (using manipulatives such as counters, number disks, dice, and so on), then moving to **Pictorial** (solving problems where pictures are involved) and finally working in the **Abstract** (where numbers represent symbolic values). Through the process, students learn numerous strategies to work with numbers and build conceptual understanding. **(CPA)**

## Procedural knowledge and Conceptual knowledge

For decades, the major emphasis in school mathematics was on procedural knowledge, or what is now referred to as procedural fluency. Rote learning was the norm, with little attention paid to understanding of mathematical concepts. Rote learning is not the answer in mathematics, especially when students do not understand the mathematics. In recent years, major efforts have been made to focus on what is necessary for students to learn mathematics, what it means for a student to be mathematically proficient. The National Research Council (2001) set forth in its document *Adding It Up: Helping Children Learn Mathematics* a list of five strands, which includes conceptual understanding. The strands are intertwined and include the notions suggested by NCTM in its Learning Principle. To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations
- Procedural fluency: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- Strategic competence: ability to formulate, represent, and solve mathematical problems
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification
- Productive disposition: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

As we begin to more fully develop the idea of conceptual understanding and provide examples of its meaning, note that equilibrium must be sustained. All five strands are crucial for students to understand and use mathematics. Conceptual understanding allows a student to apply and possibly adapt some acquired mathematical ideas to new situations.

## Following the C-P-A Approach

How can we as teachers help our students to find the meaning in the math? Research by Jerome Bruner (Bruner, 2000) states that instructional strategies build understanding for students when they move from the Concrete (manipulatives) to the pictorial (visual models or drawings) to the abstract (symbols).

At each stage in the C-P-A approach, small group and partner work is important for facilitating language growth in mathematics. Mathematical learning shouldn't happen in isolation. Interaction with other students provides opportunities for children to explain and justify their work, which is a much higher level of thinking than just "getting the answer". The teacher begins by modeling and thinking aloud about a strategy (I do), students then practice with the teacher (We do), students practice with small groups (We do) and partners (We do), and finally students practice independently (You do). "I do, we do, we do, we do, you do" method.

Strong emphasis on place value throughout lessons and throughout grade levels. Place value should not be taught as a separate topic unto itself-it should be a repeating theme throughout each math lesson. Without the concept of places and values in our base ten system, computation will be difficult and long-term understanding nearly impossible.

## **Singapore Strategies**

Below is a list of strategies taken from various Singapore Math Trainings. Click on the name of the strategy to view a video explanation and example.

**1) Math Talks (Look & Talks)**

Math Talks are guided math discussions using pictures to help students develop number awareness in our everyday world.

**2) Number Bonds**

A number bond shows how a number (whole) is made up of parts. Number bonds let children see the inverse relationship between addition and subtraction.

**3) Number Bracelets**

These bracelets made of pipe cleaners and beads reinforce the commutative property of addition as well as all of the combinations to create a number.

**4) Ten Frames**

Ten frames are rectangular frames for placing counters to illustrate the numbers 1 through 10. Arranging counters in different ways on the ten-frame prompts students to form mental images of the numbers represented.

**5) Making Ten Using Ten Frames**

Set up counters on a ten frame for the first addend in an addition problem. Then set up the counters for the second addend either on an additional ten frame or on the table. Move the counters up to make ten, which will make an easier addition problem consisting of ten plus a number.

**6) Branch Method**

A technique to help break down or decompose whole numbers into parts that are more easily manageable is called branching. After students practice breaking numbers apart into place value groupings, teach them to add and subtract by place value. The goal with branching is for students to break numbers into place value groupings and then do the operation with those place value groups.

**7) Converting Units Using the Branch Method and Part-Whole Thinking**

Just like you can use the branch method for breaking up whole numbers into their place value parts, you can also use it to break up units and convert them to

the next bigger unit (ex. minutes to hours, ounces to pounds, inches to feet, quarts to gallons, and even fractions to whole numbers).

### 8) **Place Value Disks**

Place value disks are math tools that students use to represent numbers. They are intended to be used when students understand the value of each disk. Unlike base ten tools, these disks are not proportional – their physical size isn't proportional to the quantity they represent, which makes them more abstract. These disks can be used for all of the following:

- a. **Identifying Numbers**
- b. **Addition/Subtraction**
- c. **Multiplication/Division**

### 9) **Place Value Strips**

Place value strips are another tool that students can use to represent numbers. These strips can help you compose and decompose numbers and are helpful with operations and expanded notation. This math tool helps students visualize the value of the digits within numbers.

### 10) **Compensation (addition)**

In an addition problem, you can round one or more of the numbers to numbers that are easier to work with, then compensate. For example:  $19 + 4$  becomes  $20 + 3 = 23$ . You add a number to one addend and subtract the same number from the other addend.

### 11) **Compensation (subtraction)**

In a subtraction problem, you can transform the problem into an equivalent problem that is easier. For example:  $456 - 98$  becomes  $458 - 100 = 358$  or  $500 - 56$  becomes  $499 - 55 = 444$ . You can add or subtract the same number to the minuend and the subtrahend and the difference stays the same.

### 12) **Left-to-Right Addition**

With left-to-right addition, you take your expanded parts and add them together, starting at the left and moving to the right.

### 13) **Vertical Addition**

With this strategy, instead of adding columns, moving from right to left, and regrouping, you add each column separately and write it down below. Then you add the partial sums together.

### 14) **Distributive Property for Multiplication**

To multiply a 1-digit factor by a multi-digit factor, use the distributive property. First, break the multi-digit factor into expanded form. Then multiply each place value separately by the 1-digit factor. Finally, add the partial products.

### **15) Area Model or Partial Product Method**

This model uses a box design, where factors are broken into place value groupings and written outside the box. Then each grouping is multiplied separately and added together to get a final product.

### **16) Distributive Property for Division**

You can use the distributive property with a multi-digit division problem when you have a single-digit divisor. You distribute your dividend into two or more friendlier parts. Both parts need to be a multiple of the divisor or equally divisible by the divisor.

### **17) Partial Quotient Division**

Partial quotient division is similar to long division, but instead of having to be exact on the top with a quotient, you make partial quotients and then add them to get the final quotient. You can think of it as successive approximations or estimates. The quotient is built through vertical steps that are added together.

### **18) Model Drawing (Bar Diagrams)**

Model drawing is a visual problem-solving tool that helps students solve arithmetic and algebraic word problems. Here are the steps to the model drawing method:

- 1) Read the problem.
- 2) Rewrite the question as a complete sentence leaving space for the answer.
- 3) Identify the "who" and the "what."
- 4) Draw a unit bar to model each variable.
- 5) Adjust your unit bars to match your information. Fill in your question mark.
- 6) Work your computation.
- 7) Write the answer in your sentence.